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Turbulent Prandtl number in circular pipes

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NOMENCLATURE

- c_p constant pressure specific heat [J kg^{-1} °C^{-1}]
- D pipe diameter [m]
- k thermal conductivity [W m^{-1} °C^{-1}]
- Nu Nusselt number, hD/k
- p pressure [N m^{-2}]
- Pr Prandtl number, $\mu c_p/k$
- q heat flux [W m^{-2}]
- r distance from pipe centerline [m]
- R pipe radius [m]
- Re Reynolds number, $\rho u_b D/\mu$
- t mean temperature [°C]
- u mean streamwise velocity component [m s^{-1}]
- u' velocity fluctuation in direction of mean flow [m s^{-1}]
- u^+ mean velocity scaled using friction velocity, $u/\sqrt{(\tau_\omega/\rho_\omega)}$
- v mean cross-stream velocity component [m s^{-1}]
- v' cross-stream velocity fluctuation [m s^{-1}]
- x streamwise distance [m]
- y radial distance from pipe wall [m]
- y^+ non-dimensional distance from pipe wall, $y\sqrt{(\tau_\omega/\rho_\omega)}/(\mu_\omega/\rho_\omega)$.

Greek symbols
- κ Von-Karman's constant
- μ viscosity of fluid [N s m^{-2}]
- ρ density of fluid [kg m^{-3}]
- τ shear stress [N m^{-2}].

Subscript
- t indicates turbulence quantity.

INTRODUCTION

THIS note is concerned with the turbulent Prandtl number with a view towards meeting the needs of numerical investigators. Reynolds [1] has examined more than 30 different ways of predicting the turbulent Prandtl numbers. The existing procedures range from purely empirical to formal analyses based on the Reynolds stress equation. The formal analyses are available only for the case of weak decaying turbulence. The empirical formulations vary from those containing several adjustable constants and several parameters such as the position within the flow to the simplest expressing the turbulent Prandtl number as a function of the molecular Prandtl number only, e.g. the formula due to Graber [2]

$$Pr_t^{-1} = 0.91 + 0.13Pr^{0.545}; \quad 0.7 < Pr < 100. \tag{1}$$

This last type of proposal which amounts to using one constant value of Pr_t for one particular fluid has found maximum use in numerical investigations. Elsewhere [3] one of the authors has shown that even in a highly variable property fluid (supercritical carbon dioxide) this practice leads to the best results. However, even in this approach the difficulty is that no definite proposal is available for the values of Pr_t for fluids whose Prandtl number Pr differs markedly from unity. Graber's formula is valid only in the range $0.7 < Pr < 100$. There is a need to extend this range. The turbulent Prandtl number is an extremely difficult quantity to measure experimentally. The alternative approach is to deduce its value from experimental measurements of bulk quantities such as Nusselt number which are more easily measured. This is the approach followed in the present work. The technique adopted here is to start from established values of Nusselt number in smooth pipes and deduce the value of the turbulent Prandtl number to satisfy this result. The question whether these values of Pr_t will be applicable in other geometries cannot be answered at this stage. The turbulent Prandtl number has been considered in the present work for a wide range of Prandtl numbers ($1.0 < Pr < 12\,500$) and Reynolds numbers ($10^4 < Re < 10^6$) in circular pipes.

ANALYSIS

Consider a constant property flow through a smooth circular pipe far from the inlet. The pipe is provided with uniform heating at the wall. The influence of buoyancy and viscous dissipation is regarded as negligible. The problem at hand, is, when the Nusselt number Nu and $\overline{u'v'}$ are known for a particular fluid, what is the value of Pr_t? This deduction requires the use of conservation principles. The governing equations in axisymmetric form can be written as

Continuity

$$\frac{\partial}{\partial x}(\rho ur) + \frac{\partial}{\partial y}(\rho vr) = 0 \tag{2}$$

Momentum

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial y}\left[r\left(\mu\frac{\partial u}{\partial y} - \rho\overline{u'v'}\right)\right] \tag{3}$$

Energy

$$c_p\rho u\frac{\partial t}{\partial x} + c_p\rho v\frac{\partial t}{\partial y} = \frac{1}{r}\frac{\partial}{\partial y}\left[r\left(k\frac{\partial t}{\partial y} - c_p\rho\overline{v't'}\right)\right] \tag{4}$$

where

$$\overline{v't'} = \frac{1}{Pr_t}\cdot\frac{\overline{u'v'}}{\partial u/\partial y}\cdot\frac{\partial t}{\partial y}.$$

A conventional model based on the Van-Driest [4] modification of Prandtl's hypothesis is used. It was found during computations that simply using Van-Driest's hypothesis leads to an error in the predicted velocity profiles near the core of the pipe. The use of an additional multiplying factor as described in ref. [3] removes this inaccuracy also. This additional factor has emerged as a result of studies on a two-equation model of turbulence, the momentum correlation then becomes

$$-\overline{u'v'} = k^2y^2[1-\exp(-y^+/A^+)]^2 \times \left(\frac{\partial u}{\partial y}\right)^2 \Big/ \left(1+\frac{3y}{R}+\frac{3y^2}{R^2}\right) \quad (5)$$

where the constant A^+ has the value 28. For fully developed pipe flows the continuity and momentum equations can be reduced to a first-order differential equation. The energy equation cannot be reduced to a first-order differential equation since dt/dx is not equal to zero. The integration was carried out by an explicit finite-difference procedure assuming the pipe diameter (0.04 m), wall heat flux (1000 W m^2) and wall temperature (variable). The final non-dimensional results obtained are independent of these parameters but these have to be specified for integration. The wall shear stress was also prescribed for each integration. The Reynolds number is varied by varying the input wall shear stress. The convergence and grid independence of the results depends both on the number of grid points and the spacing of the first node away from the wall. Both of these were very carefully ascertained to ensure accuracy of the calculated results.

The numerical calculations for heat transfer are executed by supplying assumed values of Pr_t for a particular fluid. These are varied until the required Nusselt number is achieved. Thus an element of iteration is necessary to deduce the values of Pr_t from known Nusselt number values. The Reynolds and Nusselt numbers are found from the bulk velocity and the bulk temperature that result from the numerically determined velocity and temperature profiles. The values of the Nusselt number supplied must be accurate so as to ensure accuracy in the derived value of Pr_t. An equation due to Petukhov and Popov [5] is reported to correlate data reported in nine different experimental studies to within 6%. Another equation derived independently by Notter and Sleicher [6] (later modified by Sleicher and Rouse [7]) agrees closely with Petukhov's correlation and may be used as an alternative especially at high Prandtl numbers.

RESULTS

Based on the numerical solution an iterative deduction of the turbulent Prandtl number was made at a Reynolds number of 100 000 for a wide range of fluids ($1.0 < Pr < 12\,500$). These results are summarized in Table 1. The table also indicates the fluids that were selected to achieve a range of Prandtl numbers. It was found that approximately a 1% change in the turbulent Prandtl number leads to a 0.6% change in the Nusselt number at low Prandtl numbers and this change decreases to 0.4% at high Prandtl numbers ($Pr \simeq 10^4$). This is well within the accuracy of the Nusselt numbers used. Therefore, the turbulent Prandtl numbers were ascertained up to the second decimal place only. To test if the calculated Pr_t are applicable at other Reynolds numbers also, calculations were made for a wide range of Reynolds numbers using the turbulent Prandtl number values of Table 1. These calculations are illustrated in Fig. 1. It can be deduced that the turbulent Prandtl number appears to be independent of the Reynolds numbers. The slight differences exhibited especially at high Reynolds numbers are well within the accuracy of Petukhov's correlation (6%) and Notter and Sliecher's correlation (10%), therefore, at this point no purpose would be served by making the turbulent Prandtl number dependent on the Reynolds number. From Fig. 1 it can be seen that a constant value of Pr_t for one particular Pr is fairly successful in predicting the Nusselt number and a more complicated practice does not appear to be necessary at least in pipe flows.

The turbulent Prandtl numbers calculated by the present procedure are compared with Graber's proposal in Fig. 2. Graber's proposal is applicable only up to $Pr = 100$. An attempt was made to correlate the values of turbulent Prandtl number in the form of a simple algebraic equation. It was found that these values can be correlated by the following set of equations

$$\begin{aligned} Pr_t &= 1.01 - 0.09Pr^{0.36}; && 1 < Pr < 145 \\ Pr_t &= 1.01 - 0.25\log Pr; && 145 < Pr < 1800 \quad (6) \\ Pr_t &= 0.99 - 0.44(\log Pr)^{1/2}; && 1800 < Pr < 12\,500. \end{aligned}$$

These three equations agree to within 1% in most cases and to within 4% for the worst case with the values of Table 1.

Although the values of Pr_t in the present work have been ascertained on the basis of a specific mixing length model, these should be of use in other numerical studies based on other turbulence models as well, such as the multi-equation models.

The present results should also be of use during development of theoretical formulations for turbulent Prandtl number as a source of comparison.

Table 1. Turbulent Prandtl numbers in circular pipes

Fluid	Pr	Pr_t	Fluid	Pr	Pr_t
Water, 180°C	1.0	0.92	Ethylene glycol, 16°C	252	0.41
Water, 100°C	1.7	0.89	Ethylene glycol, 12°C	321	0.38
Water, 80°C	2.2	0.88	Ethylene glycol, 8°C	407	0.35
Water, 60°C	3.0	0.86	Ethylene glycol, 6°C	458	0.34
Water, 40°C	4.3	0.85	Ethylene glycol, 4°C	506	0.33
Water, 20°C	7.0	0.83	Ethylene glycol, 2°C	557	0.32
Water, 5°C	11.2	0.81	Ethylene glycol, 0°C	617	0.31
Carbon dioxide, 25°C	15.0	0.78	Engine oil, 60°C	1050	0.25
Ethylene glycol, 100°C	22.4	0.74	Glycerine, 50°C	1630	0.20
Ethylene glycol, 80°C	32.4	0.70	Glycerine, 45°C	1834	0.19
Ethylene glycol, 70°C	41.9	0.66	Glycerine, 40°C	2450	0.18
Ethylene glycol, 60°C	51.0	0.64	Glycerine, 38°C	2736	0.17
Ethylene glycol, 50°C	67.2	0.60	Glycerine, 34°C	3742	0.16
Ethylene glycol, 40°C	93.0	0.55	Glycerine, 30°C	5380	0.14
Ethylene glycol, 36°C	106.5	0.53	Glycerine, 28°C	6430	0.13
Ethylene glycol, 32°C	123.6	0.51	Glycerine, 26°C	7679	0.122
Ethylene glycol, 28°C	142.8	0.49	Glycerine, 24°C	9189	0.115
Ethylene glycol, 24°C	169.7	0.48	Glycerine, 22°C	10 755	0.109
Ethylene glycol, 20°C	204.0	0.45	Glycerine, 20°C	12 500	0.100

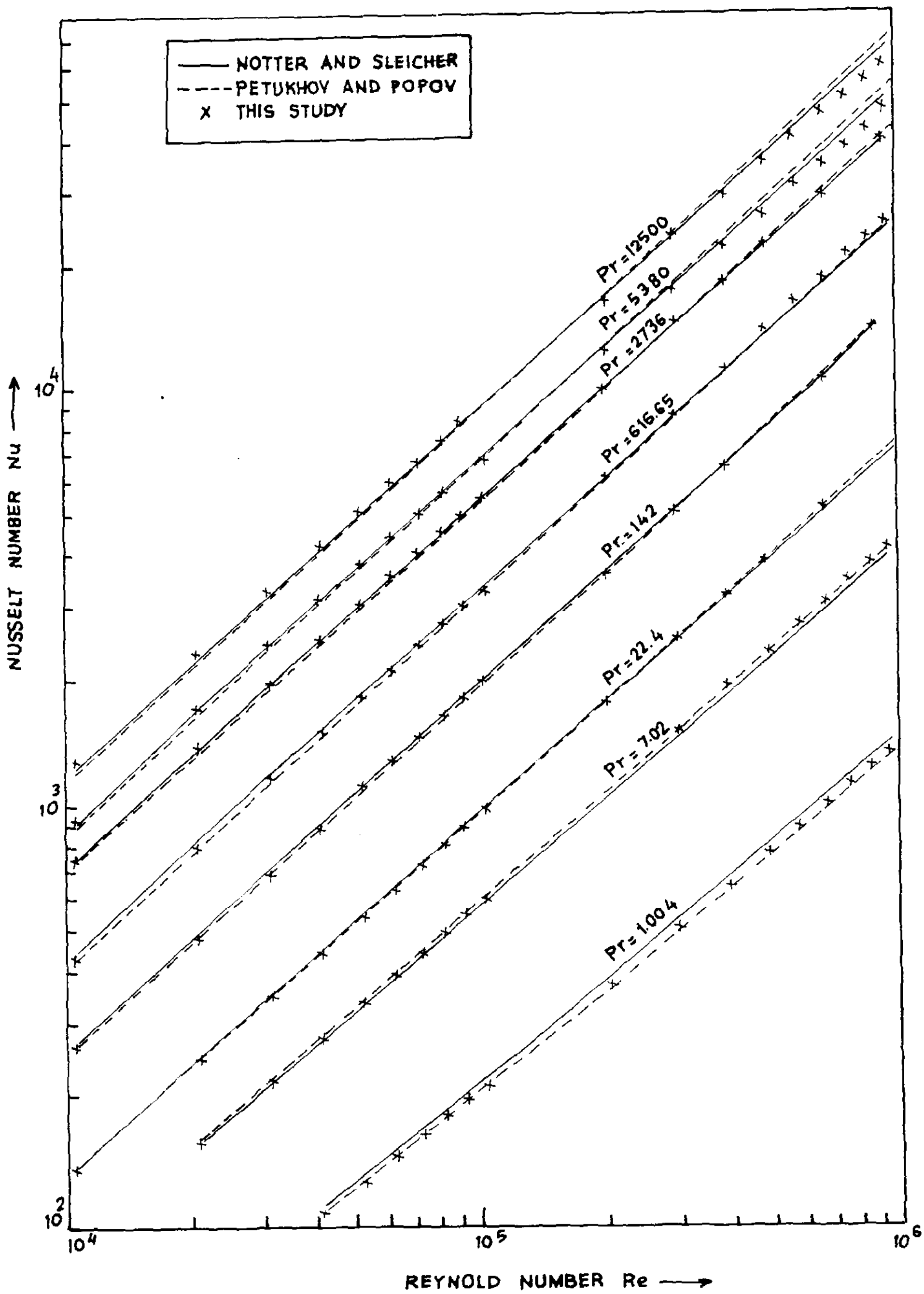

FIG. 1. Predicted and correlated Nusselt numbers in circular pipes.

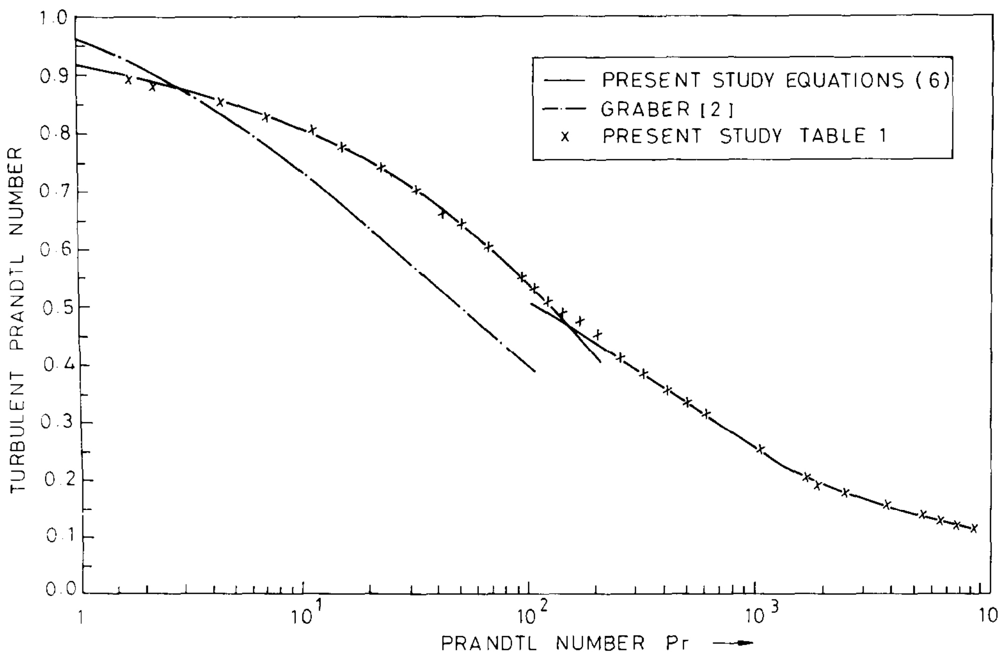

FIG. 2. Turbulent Prandtl number in the range $1 < Pr < 12\,500$.

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Determination of boiler furnace heat flux

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NOMENCLATURE

D	tube outside diameter [mm]
F	view factor
h	film conductance [kcal h^{-1} m^{-2} °C^{-1}]
p	pressure [kg cm^{-2}]
q	heat flux [kcal h^{-1} m^{-2}]
T	temperature [°C].

Greek symbols

β_1, β_2	view angles [rad]
θ	angle [rad].

Subscripts

o	tube outside
p	projected.